PROCEDURES FOR SMOOTHING AND FILTERING GEOPHYSICAL ANOMALIES TO EVALUATE GEOLOGICAL SOURCES

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Abstract. Following the acquisition, geological / geophysical observations are evaluated and assigned to geological formations. Mapping is used to compile boundary and orientation information into a 3D map showing the geometric arrangement of the boundaries between formations. The measurements of the physical properties are assigned to the formations, and the statistical methods are then used to determine the characteristics of these properties at the scale of the elements used in the inversion phase. Smoothing and filtering procedures for geophysical anomalies are very important for assessing the size and depth of geological sources. This paper presents some methods used in this regard. It presents an extension of the moving average method for 2D / 3D cases, the trend surfaces for determining the regional character of geophysical anomalies as well as comparisons with Fourier spectral methods and analytical continuations in the upper half-space.

Keywords: moving average, analytical continuations in the upper half-space, trend polynomial surfaces, spectral analysis.

Rezumat. Proceduri de netezirea și filtrarea anomaliilor geofizice pentru evaluarea surselor geologice. În urma achiziției, observațiile geologice/geofizice sunt evaluate și atribuite formațiunilor geologice. Maparea este utilizată pentru a compila informațiile de limită și orientare într-o hartă 3D care arată aranjarea geometrică a granițelor dintre formațiuni. Măsurătorile proprietăților fizice sunt atribuite formațiunilor, iar metodele statistice sunt apoi utilizate pentru a determina caracteristicile acestor proprietăți la scara elementelor utilizate în faza de inversare. Procedurile de netezirea și filtrarea anomaliilor geofizice sunt foarte importante pentru evaluarea dimensiunilor și adâncimii surselor geologice. În această lucrare sunt prezentate câteva metode utilizate în acest sens. Prezintă o extensie a metodei de mediere mobilă pentru cazuri 2D/3D, suprafețele de tendință pentru determinarea caracterului regional al anomaliilor geofizice precum și comparații cu metode spectrale Fourier și continuări analitice în semispațiul superior.

Cuvinte cheie: medierea mobilă, continuări analitice în semispațiul superior, suprafețe polinomiale de tendință, analize spectrale.

INTRODUCTION

Moving average analysis with windows of different dimensions and the surfaces of polynomial tendencies of different degrees contributes to the recognition, isolation and measurement of trends that can be represented by surfaces, thus achieving a separation of regional variations and local variations. This separation is achieved by adjusting the trend function at different values.

Geophysical data measured at the surface of the land include the weighted effect of all existing geological formations starting from the surface (with the highest weight) and up to great depths (with the lowest weight) depending on the method used.

Thus the gravimetric data measured at the surface represent the weighted effect of the density of the basement layers, from the surface to depths of tens of km (Crust - upper Mantle limit). Also, the magnetic data represend the magnetization effects of the rocks from the surface to the temperature limit at which the rocks lose their magnetic properties (Curie point).

The introduction of geophysical data sets into 3D modeling programs requires their filtering according to the depth of the model we want to make.

The trend surfaces have the great advantage that the effect of regional anomalies can be expressed as analytic functions.

These trend surfaces allow subsequent mathematical processing and interesting generalizations, with the great advantage of working with polynomial functions compared with the original discrete data.

The results of the presented methods can be compared with the Fourier spectral methods and the analytical continuations in the upper half-space, being able to establish even some equivalents between them.

METHODOLOGIES

a) Moving average windows

The simplest form of smoothing is the "moving average" which simply replaces each data value with the average of neighboring values. To avoid shifting the data, it is best to average the same number of values before and after where the average is being calculated.

For the study of time series, moving average windows is:

$$\overline{x}[i] = \frac{1}{2N+1}\sum_{n=-N}^{N} x[i+n] \qquad (1)$$

For the study of parameters in grid surface in two dimensions (2D), the moving average with a rectangular windows of size (2L+1)(2M+1) is:

$\overline{x}(i,j) = \frac{1}{(2L+1)(2M+1)} \sum_{l=-L}^{L} \sum_{m=-M}^{M} x(i+l, j+m)$ (2)

For the study of parameters in grid with three dimensions (3D), the moving average with a parallelepiped mobile window of size (2M+1)(2N+1)(2N+1) is:

$\overline{x}(i,j,k) = \frac{1}{(2L+1)(2M+1)(2N+1)} \sum_{l=-L}^{L} \sum_{m=-M}^{M} \sum_{n=-N}^{N} x(i+l, j+m, k+n)$ (3)

b) Trend polynomial surfaces

The analysis of polynomial trends is an old method, based on the smallest squares' method used for the smoothing of geological data, which many authors have utilized, such as ASIMOPOLOS N.S. (2017), ASIMOPOLOS L. & ASIMOPOLOS N.-S. (2018), FARHANG-BOROUJENY (2013), HARBAUGH (1972), KARAKUS et al. (2011) and UNWIN (1978). Also, these methods are exemplified in various applications, like POULARIKAS & RAMADAN (2006) and VISARION (1998).

Trend analysis is part of the field of regression analysis satisfying the smallest squares criterion.

The difference between the calculated value of the trend surface at a certain point and the value observed at that point is the residual value. The sum of the squares of these residual values must be minimal according to the smallest squares criterion. If the trend area is considered to be a regional or large-scale component, then the residual value should be considered as the local or small-scale component.

Removing the regional trend has the effect of highlighting local components represented by residual values. The principles of surface trend analysis and surface hyperspace apply to surfaces and hyperspace in any number of dimensions.

Polynomial trend surfaces analysis contributes to the recognition, isolation and measurement of trends that can be calculated and represented by analytical equations, thus achieving a separation in regional and local variations.

The anomaly separation operation consists in determining the number of sources, the characteristics of each source (depth, shape, and dimensions) so as to result in a cumulative total anomaly, measured at the Earth's surface. This separation has to be done in the context of the fundamental ambiguity of geophysical information, based on the cause-effect ratio.

A surface occupying a three-dimensional space is a mathematical function with a dependent variable and two independent variables.

In the most general case of space $\mathbf{R^n}$, we choose **(n-1)** independent variables and **1** dependent variable that can be expressed by the function $\mathbf{x_n = f(x_1, x_2, ..., x_{n-1})}$, representing the expression of a hyperspace. From the **n** dimensions of space $\mathbf{R^n}$, we can choose up to **3** spatial dimensions (latitude, longitude and altitude) and the other **n-4** independent variables can be represented by parameters that have a causal connection with the dependent variable that we analyse by the hyperspace tendency method.

The criterion of the smallest squares, in its general form, can be expressed succinctly by:

$\sum(X_{obs} - X_{tend})^2 = minimum$, where $\mathbf{X_{obs}}$ (respectively $\mathbf{X_{tend}}$) represent the set of observed values (respectively calculated by trend analysis).

The simplest example of a trending function is the equation of a straight line:

y* = *A* + *Bx, where ***x*** is an independent variable, ***y*** is dependent variable and constants ***A*** and ***B*** are calculated so that the sum of deviations is as small as possible.

The simplest example of a trending surface is the equation of a plane:

$z_{tend} = A + Bx + Cy$, where ***x*** and ***y*** are independent variables, ***z*** is a dependent variable and constants ***A***, ***B*** and ***C*** are calculated so that the sum of deviations is minimal. The deviation at a given point is the difference between the observed value and the calculated value and can be expressed by: **deviation** $= \mathbf{z_{obs} - z_{tend}}$. It follows that: **deviation** = $\mathbf{z_{obs} - A - Bx - Cy}$.

Expressing the sum of the squares of the deviations with the function ***Φ(A,B,C)*** we obtain: $\Phi(A,B,C) = \sum_{i=1}^{n}(z_i - A - Bx_i - Cy_i)^2$, where z_i are the values observed at each coordinate point (x_i, y_i) and ***n*** is the total number of observation points.

If ***Φ(A,B,C)*** must be minimized then it is necessary to: $\frac{\delta\Phi}{\delta A} = \frac{\delta\Phi}{\delta B} = \frac{\delta\Phi}{\delta C} = 0$ (4)

from which it follows that:

$$\frac{\delta F}{\delta A} = \sum_{i=1}^{n} 2(z_i - A - Bx_i - Cy_i)(-1) = 0$$

$$\frac{\delta F}{\delta B} = \sum_{i=1}^{n} 2(z_i - A - Bx_i - Cy_i)(-x) = 0 \quad (5)$$

$$\frac{\delta F}{\delta C} = \sum_{i=1}^{n} 2(z_i - A - Bx_i - Cy_i)(-y) = 0$$

By multiplying each expression and summing the individual terms of these three equations, we obtain the system:

$$-\sum_{i=1}^{n} z_i + A \cdot n + B\sum_{i=1}^{n} x_i + C\sum_{i=1}^{n} y_i = 0$$

$$-\sum_{i=1}^{n} z_i x_i + A\sum_{i=1}^{n} x_i + B\sum_{i=1}^{n} x_i^2 + C\sum_{i=1}^{n} x_i y_i = 0 \quad (6)$$

$$-\sum_{i=1}^{n} z_i y_i + A\sum_{i=1}^{n} y_i + B\sum_{i=1}^{n} x_i y_i + C\sum_{i=1}^{n} y_i^2 = 0$$

This system of three equations with three unknowns (***A, B*** and ***C***) can be solved by the matrix equation:

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} n & \sum x & \sum y \\ \sum x & \sum x^2 & \sum xy \\ \sum y & \sum xy & \sum y^2 \end{bmatrix}^{-1} \times \begin{bmatrix} \sum z \\ \sum zx \\ \sum zy \end{bmatrix} \qquad (7)$$

Trend surfaces of the second degree or higher are calculated similarly.

Following the same procedure, we have developed programs for the following types of surfaces:

Trend polynomial surface of second degree with the equation:

$\boldsymbol{Z = A+BX+CY+DX^2+EXY+FY^2}$ (8)

b1) Trend cubic surfaces

$\boldsymbol{Z=A+BX+CY+DX^2+EXY+FY^2+GX^3+HX^2Y+IXY^2+JY^3}$ (9)

$\boldsymbol{\Phi(A,B,C,D,E,F,G,H,I,J)=\Sigma(Z- A-BX-CY-DX^2-EXY-FY^2-GX^3-HX^2Y-IXY^2-JY^3)^2}$ (10)

After differentiation of this function we obtain next system with 10 equations (11):

$$\frac{\partial\Phi}{\partial A} = \sum\left(-2\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial B} = \sum\left(-2X\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial C} = \sum\left(-2Y\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial D} = \sum\left(-2X^2\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial E} = \sum\left(-2XY\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial F} = \sum\left(-2Y^2\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial G} = \sum\left(-2X^3\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial H} = \sum\left(-2X^2Y\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial I} = \sum\left(-2XY^2\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0$$
$$\frac{\partial\Phi}{\partial J} = \sum\left(-2Y^3\left(Z - A - BX - CY - DX^2 - EXY - FY^2 - GX^3 - HX^2Y - IXY^2 - JY^3\right)\right) = 0 \cdots$$

From (11), we obtain the following system (12):

$$An + B\sum X + C\sum Y + D\sum X^2 + E\sum XY + F\sum Y^2 + G\sum X^3 + H\sum X^2Y + I\sum XY^2 + J\sum Y^3 = \sum Z$$
$$A\sum X + B\sum X^2 + C\sum XY + D\sum X^3 + E\sum X^2Y + F\sum XY^2 + G\sum X^4 + H\sum X^3Y + I\sum X^2Y^2 + J\sum XY^3 = \sum XZ$$
$$A\sum Y + B\sum XY + C\sum Y^2 + D\sum X^2Y + E\sum XY^2 + F\sum Y^3 + G\sum X^3Y + H\sum X^2Y^2 + I\sum XY^3 + J\sum Y^4 = \sum YZ$$
$$A\sum X^2 + B\sum X^3 + C\sum X^2Y + D\sum X^4 + E\sum X^3Y + F\sum X^2Y^2 + G\sum X^5 + H\sum X^4Y + I\sum X^3Y^2 + J\sum X^2Y^3 = \sum X^2Z$$
$$A\sum XY + B\sum X^2Y + C\sum XY^2 + D\sum X^3Y + E\sum X^2Y^2 + F\sum XY^3 + G\sum X^4Y + H\sum X^3Y^2 + I\sum X^2Y^3 + J\sum XY^4 = \sum XYZ$$
$$A\sum Y^2 + B\sum XY^2 + C\sum Y^3 + D\sum X^2Y^2 + E\sum XY^3 + F\sum Y^4 + G\sum X^3Y^2 + H\sum X^2Y^3 + I\sum XY^4 + J\sum Y^5 = \sum Y^2Z$$
$$A\sum X^3 + B\sum X^4 + C\sum X^3Y + D\sum X^5 + E\sum X^4Y + F\sum X^3Y^2 + G\sum X^6 + H\sum X^5Y + I\sum X^4Y^2 + J\sum X^3Y^3 = \sum X^3Z$$
$$A\sum X^2Y + B\sum X^3Y + C\sum X^2Y^2 + D\sum X^4Y + E\sum X^3Y^2 + F\sum X^2Y^3 + G\sum X^5Y + H\sum X^4Y^2 + I\sum X^3Y^3 + J\sum X^2Y^4 = \sum X^2YZ$$
$$A\sum XY^2 + B\sum X^2Y^2 + C\sum XY^3 + D\sum X^3Y^2 + E\sum X^2Y^3 + F\sum XY^4 + G\sum X^4Y^2 + H\sum X^3Y^3 + I\sum X^2Y^4 + J\sum XY^5 = \sum XY^2Z$$
$$A\sum Y^3 + B\sum XY^3 + C\sum Y^4 + D\sum X^2Y^3 + E\sum XY^4 + F\sum Y^5 + G\sum X^3Y^3 + H\sum X^2Y^4 + I\sum XY^5 + J\sum Y^6 = \sum Y^3Z$$

From this system we calculate the coefficients: ***A, ..., J.***

b2) Trend bi-cubic surface

$\boldsymbol{Z=A+BX+CX^2+DX^3+EY+FXY+GX^2Y+HX^3Y+IY^2+JXY^2+KX^2Y^2+LX^3Y^2+MY^3+NXY^3+OX^2Y^3+PX^3Y^3}$ (13)

The final matrix equation (14) for calculation of the 16 coefficients ***A, B, ..., P*** is the following equation, where we have denoted, for simplicity of writing, by Σ x, y, z the sum $\sum_{i=1}^{n} x_{i,} y_i, z_i$ and ***n*** is the total number of points on the grid surface, whose values we have taken in the calculation of the trend surface.

$$
\begin{vmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \\ M \\ N \\ O \\ P \end{vmatrix} =
\begin{vmatrix}
n & \sum X & \sum X^2 & \sum X^3 & \sum Y & \sum XY & \sum X^2Y & \sum X^3Y & \sum Y^2 & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum Y^3 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 \\
\sum X & \sum X^2 & \sum X^3 & \sum X^4 & \sum XY & \sum X^2Y & \sum X^3Y & \sum X^4Y & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 \\
\sum X^2 & \sum X^3 & \sum X^4 & \sum X^5 & \sum X^2Y & \sum X^3Y & \sum X^4Y & \sum X^5Y & \sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 \\
\sum X^3 & \sum X^4 & \sum X^5 & \sum X^6 & \sum X^3Y & \sum X^4Y & \sum X^5Y & \sum X^6Y & \sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^6Y^2 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^6Y^3 \\
\sum Y & \sum XY & \sum X^2Y & \sum X^3Y & \sum Y^2 & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum Y^3 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum Y^4 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 \\
\sum XY & \sum X^2Y & \sum X^3Y & \sum X^4Y & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 \\
\sum X^2Y & \sum X^3Y & \sum X^4Y & \sum X^5Y & \sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 \\
\sum X^3Y & \sum X^4Y & \sum X^5Y & \sum X^6Y & \sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^6Y^2 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^6Y^3 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 & \sum X^6Y^4 \\
\sum Y^2 & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum Y^3 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum Y^4 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 & \sum Y^5 & \sum XY^5 & \sum X^2Y^5 & \sum X^3Y^5 \\
\sum XY^2 & \sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 & \sum XY^5 & \sum X^2Y^5 & \sum X^3Y^5 & \sum X^4Y^5 \\
\sum X^2Y^2 & \sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 & \sum X^2Y^5 & \sum X^3Y^5 & \sum X^4Y^5 & \sum X^5Y^5 \\
\sum X^3Y^2 & \sum X^4Y^2 & \sum X^5Y^2 & \sum X^6Y^2 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^6Y^3 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 & \sum X^6Y^4 & \sum X^3Y^5 & \sum X^4Y^5 & \sum X^5Y^5 & \sum X^6Y^5 \\
\sum Y^3 & \sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum Y^4 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 & \sum Y^5 & \sum XY^5 & \sum X^2Y^5 & \sum X^3Y^5 & \sum Y^6 & \sum XY^6 & \sum X^2Y^6 & \sum X^3Y^6 \\
\sum XY^3 & \sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum XY^4 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 & \sum XY^5 & \sum X^2Y^5 & \sum X^3Y^5 & \sum X^4Y^5 & \sum XY^6 & \sum X^2Y^6 & \sum X^3Y^6 & \sum X^4Y^6 \\
\sum X^2Y^3 & \sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^2Y^4 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 & \sum X^2Y^5 & \sum X^3Y^5 & \sum X^4Y^5 & \sum X^5Y^5 & \sum X^2Y^6 & \sum X^3Y^6 & \sum X^4Y^6 & \sum X^5Y^6 \\
\sum X^3Y^3 & \sum X^4Y^3 & \sum X^5Y^3 & \sum X^6Y^3 & \sum X^3Y^4 & \sum X^4Y^4 & \sum X^5Y^4 & \sum X^6Y^4 & \sum X^3Y^5 & \sum X^4Y^5 & \sum X^5Y^5 & \sum X^6Y^5 & \sum X^3Y^6 & \sum X^4Y^6 & \sum X^5Y^6 & \sum X^6Y^6
\end{vmatrix}^{-1} *
\begin{vmatrix} \sum Z \\ \sum XZ \\ \sum X^2Z \\ \sum X^3Z \\ \sum YZ \\ \sum XYZ \\ \sum X^2YZ \\ \sum X^3YZ \\ \sum Y^2Z \\ \sum XY^2Z \\ \sum X^2Y^2Z \\ \sum X^3Y^2Z \\ \sum Y^3Z \\ \sum XY^3Z \\ \sum X^2Y^3Z \\ \sum X^3Y^3Z \end{vmatrix}
$$

b3) Trend hyper-surfaces

In R^4 space general equation of a hyper-surface is: ***z = f(x,y,w)*** $= \sum_{i=0}^{m}\sum_{j=0}^{n}\sum_{k=0}^{p} a_{ijk}x^i y^j w^k$, (15) where ***z=f(x,y,w)*** is dependent variable depending on the independent variables ***x, y*** and ***w***. In all terms of this equation ***x*** can have maximum degree equal with ***m, y*** can have maximum degree equal with ***n*** and ***w*** can have maximum degree equal with ***p***. Coefficients $\mathbf{a_{ijk}}$ can be calculated as in the case of trend polynomial surfaces.

We have elaborated and used software for particular cases of tendency hyper-surfaces, for degree up to three.

For degree 1: $Z=A+BW+CX+DY$ (16)

For degree 2: $Z=A+BW+CX+DY+EX^2+FWY+GXY+HWX+IW^2+JY^2$ (17)

For degree 3:

$Z=A+BW+CX+DY+EX^2+FWY+GXY+HWX+IW^2+JY^2+KX^3+LW^3+MY^3+NX^2Y+OXY^2+PY^2W+QYW^2+RW^2X+SWX^2+TWXY$ (18)

CASE STUDY AND RESULTS

The Romanian terrestrial data we used are the Bouguer Anomaly Map, for the density of 2.67 g/cm^3, scale 1: 1,000,000, from the websites: https://www.igr.ro and https://bgi.obs-mip.fr and the topographic data were digitized from the topo maps of Romania at 1:25000 scale. We transcribed the coordinates from geographical coordinates to the Stereo 70 system, metric coordinates.

For the territory of Romania, Fig. 1 presents the map of residual Bouguer anomaly, after removing trend plane and tendency surfaces of order 6.

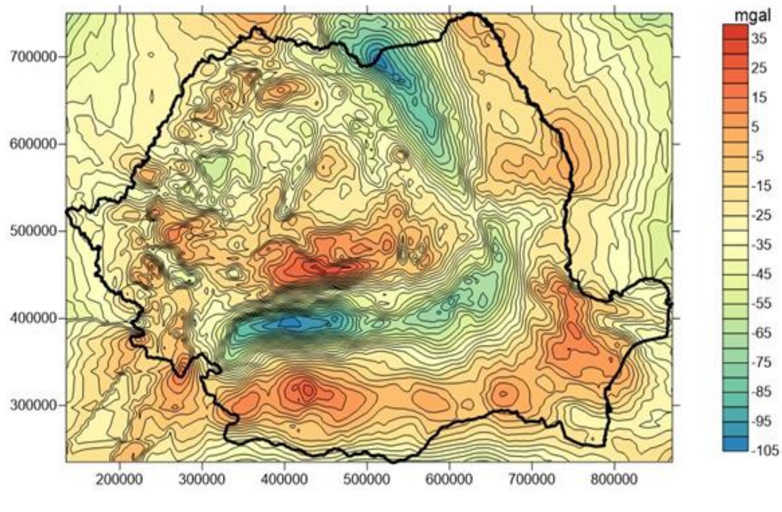

Figure 1. Map of residual Bouguer anomaly. From the values of the Bouguer anomaly, we extracted the trend plane and the trend surfaces of orders 6 (after ASIMOPOLOS N.S. 2017, with data from https://bgi.obs-mip.fr).

The equations for the trend plane and the bi-cubic tendency surface (order 6):

$z = -6.4688737669X-0.0019840798Y-0.0365902330$

$z = -890.0427063806+5.3618495844X-0.0044335874X^2-0.0000008798X^3+6.9386948354Y-0.0437772562XY +0.0000483387X^2Y-0.0000000063X^3Y-0.0154970145Y^2+0.0001026575XY^2-0.0000001351X^2Y^2+ +0.0000000000X^3Y^2+0.0000107958Y^3-0.0000000738XY^3+0.0000000001X^2Y^3+0.0000000001X^3Y^3$

Figure 2 is the map at the Earth's surface of the Bouguer anomaly and the analytical continuations in the upper half-space for several heights: 2600 m, 2800 m, 3000 m, 3500 m, 4000 m, 5000 m and 10000 m.

It is observed that, as the height at which the anomaly values are recalculated increases, they become more and more attenuated, due to the decreasing effect of gravity given by the differences in density of the geological masses in the subsoil and their distribution.

A similar result was obtained using moving average performed for different rectangular floating windows with 9 values (3 rows x 3 columns), 25 values (5 rows x 5 columns), 49 values (7 rows x 7 columns), 81 values (9 rows x 9 columns) and 121 values (11 rows x 11 columns).

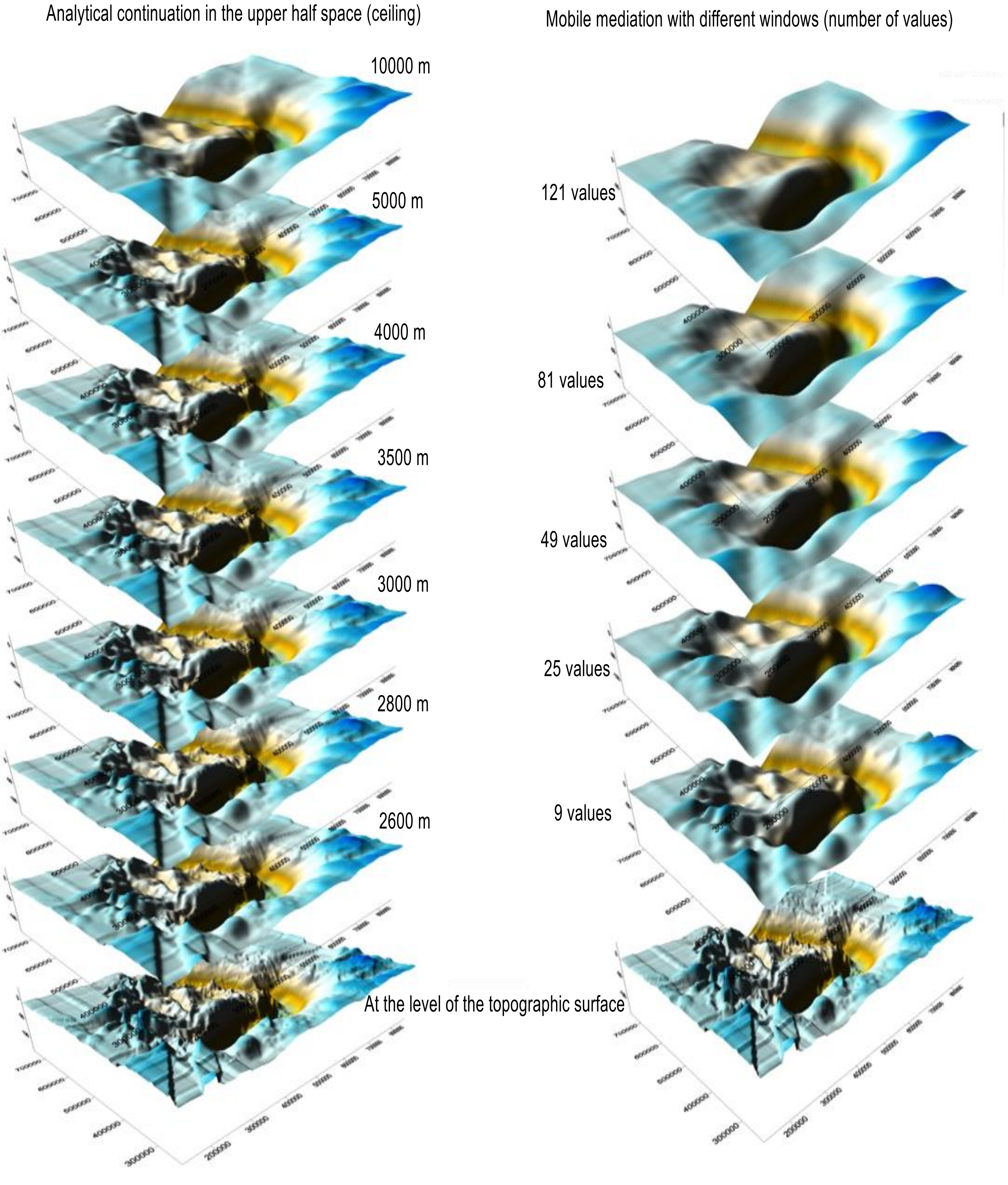

Figure 2. 2D Bouguer anomaly of Romania from the https://bgi.obs-mip.fr data; Left image- analytical continuation in the upper half-space (at different calculation heights); Right image- moving average with different square windows.

Figure 3 shows the 2D Fourier analysis of the map of Romania's Free Air anomaly from https://bgi.obs-mip.fr.

In the preliminary stage we created a program in Microsoft Excel, for the conversion of data files from Surfer format to matrix format (necessary for entering data in Matlab) for the parameters used. We then created program sequences in Matlab (https://www.mathworks.com) for 2D Fourier analysis, using the functions: 2D Fast Fourier Transform (fft2), 2D Reverse Fast Fourier Transform (ifft2), and specific parameters calculated based on these transforms: real part (real), part imaginary (imag), modulus (abs) and angle (angle).

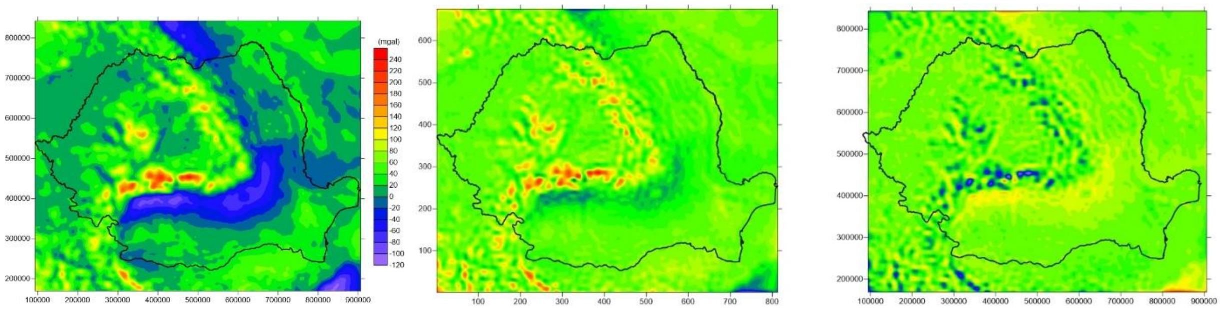

Figure 3. 2D Fourier analysis of the map of the Free Air anomaly of Romania from the https://bgi.obs-mip.fr data. Free Air anomaly map before applying FFT and IFFT (left image). Free Air anomaly map after FFT and IFFT application (middle image). Residual map showing the difference between the original map and the filtered map (right image). The color scale (mgal) is the same for all three images.

CONCLUSIONS

The procedures for smoothing and filtering geophysical anomalies to evaluate geological sources such as moving averages with windows of different dimensions and the polynomial tendencies of different degrees are particularly useful tools for separating regional effects from local ones, in the process of interpreting geophysical anomalies in geological terms.

The comparison of the latter with the results of other types of filtering such as Fourier 2D analysis, Wavelet analysis with different functions, upward and downdward analytical continuation, shows a very good correlation of the results as well as a completion of the interpretative information.

Some of the gravity anomalies that have deep causes may be masked by the presence of geological formations of a very different density compared to deep formations. Gravity anomalies caused by small-scale formations can be easily recognized in residual maps after regional trends are eliminated.

In the comparative study on the filtering of gravimetric data for the territory of Romania, we presented the algorithms and programs developed for moving average with various windows and for trend surfaces up to the 6th order,-In the comparative study on 2D Fourier analysis and analytical continuations in the upper half-space, we performed spectral analysis, deconvolution and synthesis of gravity anomaly maps. For this, we mainly used the Matlab program (signal processing) and the Surfer program to create interpolated maps and highlight details overlaid on tectonic information.

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